Carthage Mathematics Department
Course Summary for Math 322 Abstract Algebra I

1. Credits: 4 credits
2. Semesters Offered: Fall
3. Text(s): *Contemporary Abstract Algebra* by Joseph Gallian, sixth edition
4. Topics Covered:
    a. Properties of integers
    b. Modular arithmetic
    c. Mathematical induction
    d. Equivalence relations
    e. Functions
    f. Symmetries of a square and other dihedral groups
    g. Definition and examples of group
    h. Group and element properties
    i. Subgroups
    j. Cyclic groups
    k. Permutation groups
    l. Isomorphisms
    m. Cosets and Lagrange's Theorem
    n. External direct products
    o. Normal subgroups and factor groups
    p. Group homorphisms
    q. Fundamental Theorem of Finite Abelian Groups
    r. Introduction to rings
    s. Introduction to fields
5. Skills Enhanced:
    a. Technical writing: Students complete a typed final project on an application of linear algebra of 10-20 pages.
    b. Computer skills:
        1. Mathematica
        2. MS Word
    c. Oral presentations: Students give a formal presentation of 10-15 minutes on their final projects and often present problems in class.

6. Sample Syllabus:

   Chapters 0-11 and parts of Chapters 12-13

7. Miscellanea

8. Sample Assessment Questions:

Question 1: Define and give an example of each of the following terms.
(a) associativity
(b) Abelian/commutative
(c) cyclic group
(d) order of a group
(e) centralizer of an element
(f) binary operation

Question 2: Determine if the given sets and operations are or are not groups. Be sure to explain your answers.

(a) $(Z-\{0\},\bullet)$ (b) $(Q,-)$
(c) $(Z_n,+)$ (d) (D_5,o)

Question 3: Complete the following problems:

(a) In the group $(Z_{20},+)$, find:
   (i) $|\overline{5}|$
   (ii) $C(\overline{2})$
   (iii) its subgroup lattice
   (iv) the generators of this group

(b) Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G.

(c) Suppose that a belongs to a group and |a| = 5. Prove that $C(a) = C(a^3)$.

Question 4: Prove that a group of order 5 must be cyclic.

Question 5: Define and give an example of each of the following terms.
(a) permutation
(g) isomorphism
(h) kernel of a map
(i) Aut(G)
(j) injective

Question 6: Complete the following problems:

(a) Write the permutation below in disjoint cycle form and as the product of 2-cycles:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 3 & 1 & 2 & 9 & 8 & 7 & 11 & 12 & 6 & 10 & 13 & 4 & 5 \end{bmatrix}$$

(b) Find the inverse of the permutation (17832)(4659).

(c) What is the order of the permutation in (b)?

(d) Find the product of the cycles below:
(1345)(231)(453678)(432676)(23)(5472)

Question 7: Determine if the given pairs of groups are isomorphic. If the groups are isomorphic, define an isomorphism between the pair of groups.

(a) $(Q-\{0\},\bullet)$ and $(Z,+)$ (b) $(U_{12},\bullet)$ and $(Z_2 \oplus Z_2,+)$

(c) $(Z_2 \oplus Z_3,+)$ and $(Z_6,+)$ (d) $(S_3,\circ)$ and $(Z_6,+)$

Question 8: Let ϕ be the group homomorphism defined by
$\phi:(Z_{24},+)\to(Z_{20},+)$ by $\phi(n)=(n \bmod 4)*5$

(a) What is kerϕ?

(b) What is Imϕ?

(c) Apply the First Isomorphism Theorem to define an isomorphism from Z_{24}/kerϕ to Imϕ and show this isomorphism in a diagram between the original two groups.

Question 9: Complete the following problems:

(a) Let G be an abelian group. Show that the map $\alpha(g) = g^{-1}$ is an automorphism.

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(b) Prove that S_n is non-abelian for all $n \geq 3$.