

## ***The Love Prophet: A Mathematical Model for Romance***

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Love is many things all wrapped in one: mysterious, exhilarating, amusing, painful, challenging, and delightful to name a few. But one thing we can all agree on is that love is next to impossible to foresee. Perhaps the most unpredictable event in a person's life is finding someone to love for the rest of time. Wouldn't it be easier for everyone if we had a tool that could immediately tell us whether or not two people would fall in love? Surely a lot of broken hearts could be avoided by implementing this powerful "love prophet". To that end we set out to make a mathematical model of love and romance, such that given a man and woman's initial feelings for each other and the man's starting level of effort put into pursuing the woman, we can immediately tell if it is a match made in heaven, or a duo destined for divorce.

### **Setup:**

To set up our equations, we need to model the change in each variable as a function of all variables in the model. This was accomplished by drawing graphs of what the expected rates of change were and concocting functions to fit the graphs. The differential equation for each variable had three terms – one for change with respect to the man's feelings, one for change with respect to the woman's feelings, and one for change with respect to the man's effort.

Description of Variables			
$M$	Man's feelings	$\dot{M}$	Rate of change of man's feelings over time
$W$	Woman's feelings	$\dot{W}$	Rate of change of woman's feelings over time
$E$	Man's effort	$\dot{E}$	Rate of change of man's effort over time

Drawing the graphs and creating functions to fit the graphs centers on critical points of the function. For example, determining how a man's feelings changed relative to the woman's feelings required a value for  $\dot{M}$  at  $W = 0$  and  $W = 1$ . Other key points included where  $\dot{M} = 0$ . Once values at these critical points are known, we can find a sine function that connects all these dots. Following the same guidelines for each variable, we are able to create a system of differential equations that can be analyzed and studied.

Before jumping ahead to the equations, it is important to understand the reasoning behind each term. Following is a description of how each equation was derived. The corresponding graphs can be found in Appendix A.

Most men appreciate any kind of attention given to them by members of the opposite sex. As such, the man's feelings are modeled to increase as long as the woman's feelings lie somewhere between 0.25 and 1, with a negative rate of change occurring for any value below 0.25. To emphasize the notion that too much of a good thing isn't always so good, the increase in the man's feelings dwindles back to 0 as the woman's feelings approach 1. The graph relating man's feelings to his effort takes on a normal parabolic shape, with too little or too much effort having detrimental effects on his love life, and anything moderate resulting in heightened feelings for the woman. The term relating a man's feelings to itself, as with all the other terms,

serves as a corrector term keeping the man's feelings within the appropriate bounds. It represents that his feelings tend away from the extremes.

The change in the woman's feelings relative to that of the man were modeled in the exact same way as the change in the man's feelings relative to the women's feelings. The corrector term was also very similar to the aforementioned term in the man's equation. The big difference in this equation was how a woman responded to the man's effort. Since using little to no effort in pursuing a woman is often interpreted as "playing hard to get," a woman's feelings actually increase when a man's effort is close to 0. However, as his effort reaches and goes beyond 0.85, passionate pursuit turns to stalking, and the woman's feelings quickly decline.

Modeling the man's effort was rather simple. As a man's feelings increase, so does his effort to pursue his love interest. Also, when a woman's feelings fall in the low to medium category, the man increases his effort in an attempt to win over the woman's heart. When the woman's feelings are high, a man will likely become complacent, and his efforts will dwindle off. The effort equation also has an appropriate corrector term to stabilize its values.

To make things equally weighted, the corrector terms have a coefficient of 1, while the other two terms are weighted equally at 0.5. The resulting equations are as follows:

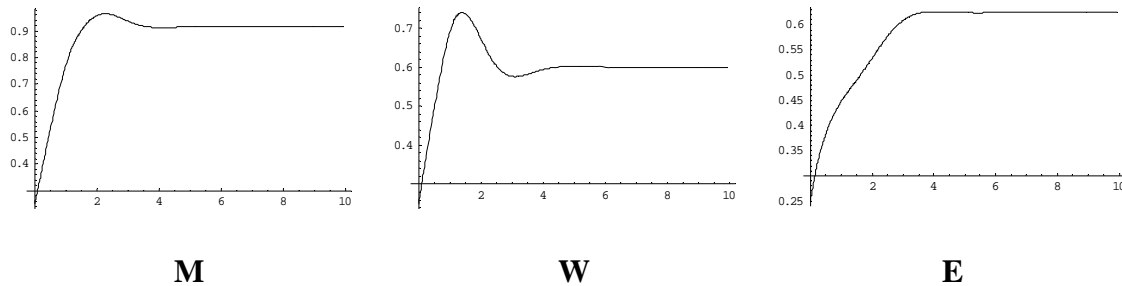
$$\begin{aligned}\dot{M} &= c_1 \left( \frac{-7}{4} M + \frac{3}{4} \right) + c_2 \sin\left(\frac{-\pi}{12} + \frac{\pi}{0.9} W^2\right) + c_3 \sin\left(\frac{-\pi}{6} + \frac{4\pi}{3} E\right) \\ \dot{W} &= c_1 \left( \frac{-3}{2} W + \frac{1}{2} \right) + c_4 \sin\left(\frac{-\pi}{12} + \frac{\pi}{0.9} M^2\right) + c_5 \sin\left(\frac{\pi}{15} + \frac{15\pi}{15} E^{0.42}\right) \\ \dot{E} &= c_1 \left( \frac{-5}{4} E + \frac{1}{4} \right) + c_6 M + c_7 \sin\left(\frac{13\pi}{12} W^{0.25}\right)\end{aligned}$$

where  $c_1 = 1$  and  $c_2 = c_3 = c_4 = c_5 = c_6 = c_7 = 0.5$ .

### **Initial Results:**

After doing several trials, it is clear that the values for M, W, and E are converging for any set of initial conditions to  $M \approx 0.916$ ,  $W \approx .601$ , and  $E \approx 0.625$ , therefore these equations

have attracting equilibrium points! Following are some graphs showing the path each variable takes as it approaches its equilibrium for the initial conditions  $M = W = E = 0.25$ .



Both graphs exhibit a strong initial onslaught of feelings followed by a small decline before evening out at the equilibrium level. This is the expected reaction for a couple entering a new relationship. Overcome by those new emotions at first, both the man and the woman settle down into their comfort zones rather quickly, with the woman experiencing the greater loss of feeling. A detailed list of initial conditions and results can be found in Appendix C.

**Continuation:**

This does not exactly satisfy the dilemma presented in the introduction, so perhaps better models are needed. We will start from scratch and create new graphs and functions. We also ditch the sine approach and turn to logarithmic functions and polynomials. The same basic reasoning is used as before, except that each variable now has a term relating it to itself independent of the corrector term. The corrector term (the second line in each equation) comes last in each equation and pulls from all three previous terms. For the effort we assume a simple linear relationship. A man's effort increases as his feelings intensify. Also, as the woman builds stronger feelings and the man becomes more comfortable in the relationship, the effort tends to fall. Much like our earlier models, effort also tends away from the extremes, explaining the negative slope in the third term of its equation. This new approach yields the following:

$$\dot{M} = c_1(3M(1-M) - 0.3) + c_2 \ln(W + 0.606) + c_3(3E(0.8 - E)) + (-0.75c_1 + (\ln(0.606) - \ln(1.606))c_2 - 1.08c_3)M + (0.3c_1 - \ln(0.606)c_2 + 0.6c_3)$$

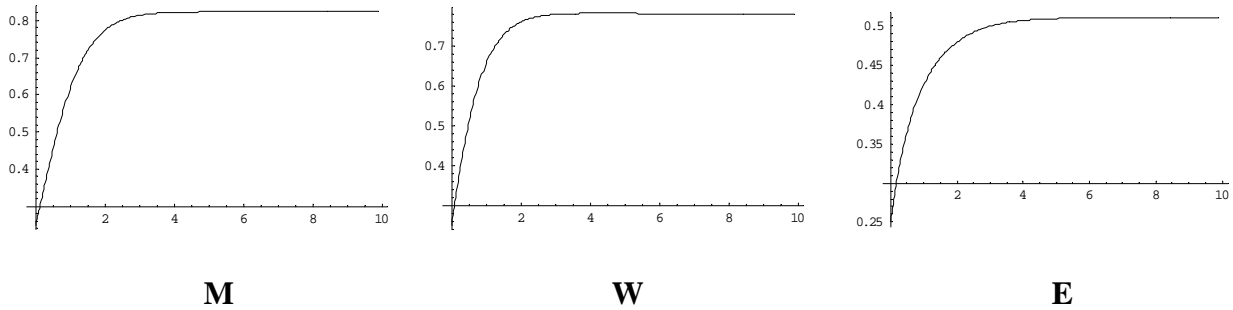
$$\dot{W} = c_4 \ln(1.5M + 0.497) + c_5 \left[ \frac{1}{3} (\ln(28W + 0.202) - 5W^2) \right] + c_6 \left[ \frac{2}{3} \sin\left(\frac{\pi-0.2}{0.75} E + 0.2\right) \right] + (-1.221c_6 - 1.121c_5 + (\ln(0.497) - \ln(1.997))c_4)W + (0.554c_6 + 0.554c_5 - \ln(0.497)c_4)$$

$$\dot{E} = c_7(M - 0.5) + c_8(0.5 - W) + c_9(0.5 - E) + (-c_7 - c_8 - c_9)E + (0.5c_7 + 0.5c_8 + 0.5c_9)$$

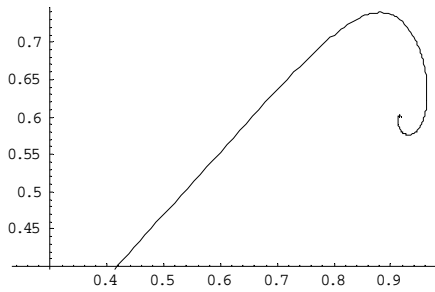
where  $c_i = \frac{1}{3}$  for  $1 \leq i \leq 9$ . Graphs are presented in Appendix B.

### Secondary Results and Interpretation:

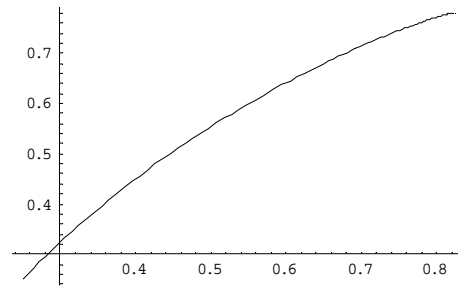
Once again, these equations have equilibrium points, this time at  $M \approx 0.823$ ,  $W \approx .780$ , and  $E \approx 0.511$ .



These graphs, again with  $M = W = E = 0.25$ , tell a slightly different story. In this model, the variables take a much smoother route to their equilibrium, foregoing any drastic falls. This provides a more desirable description of love, where a couple's feelings for each other simultaneously increase until reaching a steady, desirable level. The following graphs, showing the man's feelings along the x-axis and the woman's feelings along the y-axis, paint a vivid picture of this contrasting result from trial one and two. The second Love Prophet clearly shows a more steady and consistent relationship.



**Love Prophet I**



**Love Prophet II**

A listing of all initial conditions and their results can be found in Appendix C.

While our second trial again showed no signs of prophecy, it did reinforce some observations from the initial model. It is clear that the man's feelings tend to be stronger than the woman's feelings. This is an indication that a man will typically fall harder for a woman and be more attached in the early stages of the relationship. Also, the ideal level of effort seems to be in the middle of the spectrum, confirming that while a man must clearly be willing to put forth a solid effort, extreme actions like stalking or playing hard to get are not going to be successful in the long run.

The fact that these models reach equilibrium points also opens the door to a new question. How does modifying the weights given to each term affect the overall equilibrium levels? To answer this dilemma and gain further insight into the concept of love, we will take a look at the second Love Prophet model, individually changing each weight from  $1/3$  to 1 and analyzing the results.

### **Results and Interpretation:**

Most of the weights had a significant effect on our resulting equilibrium points. The only immaterial outcomes were seen when the women's feelings were given added weight in the equation for the man's feelings and when effort was changed in its own equation. Following is a brief philosophical and mathematical analysis of the other seven results.

When the weight for the man's feelings in his own equation is beefed up, his resulting equilibrium point diminishes considerably compared to the small drops in the woman's feelings and his effort. This implies that, barring other factors, a man has a hard time maintaining very strong feelings. While initial feelings may spike upwards, as time goes on the man tends to lose interest in the woman. This is symbolic of a man's resistance to settling down and his preference to have short relationships with several women in the hope of finding his true love. How does he know when he has found true love? According to the Love Prophet, it's when he doesn't need to work too hard to maintain a strong relationship. As more weight is put on the man's effort, it is clear that levels near 50% greatly increase the man's feelings for the woman and, likely, his desire to stay in a long relationship.

From the results received when the man's feelings are given extra weight, it follows that women love to be loved. Even though the man's effort saw a small decrease, his strong feelings correlated into a sharp increase in the woman's feelings, and clearly enhanced the state of the relationship. Women, too, seem to have a hard time prolonging a steady relationship. When more weight was given to the woman's feelings, her resulting equilibrium plummeted, forcing the man to increase his efforts, which in turn causes a reduction in his feelings. Similar to the man, the woman sees an increase in feelings when more weight is applied to the effort the man is putting forth. This means that a woman, too, prefers a moderate level of effort while being pursued. She needs enough attention to make her feel wanted, but likewise needs enough space to maintain her independence.

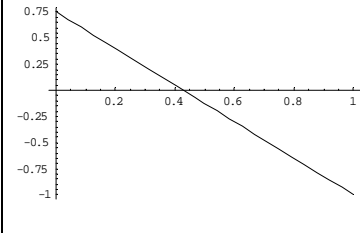
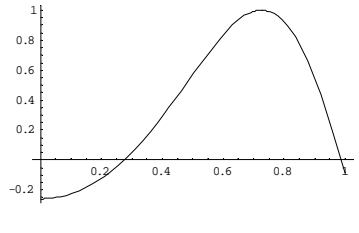
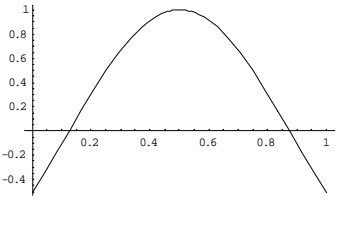
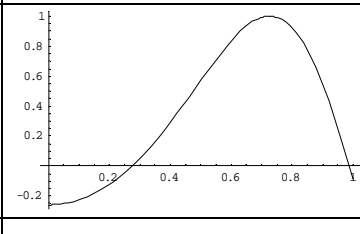
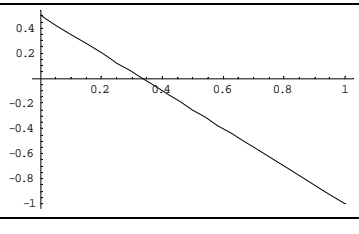
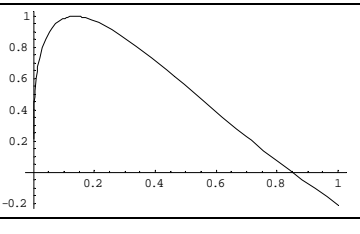
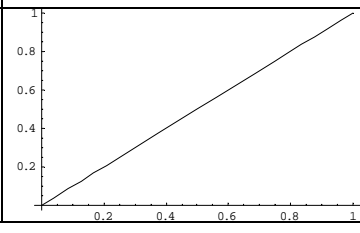
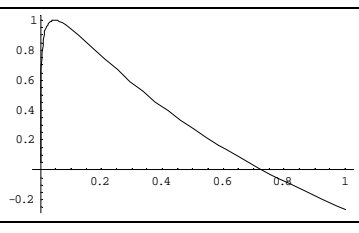
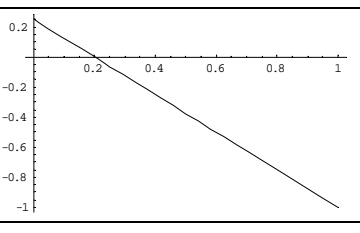
Since a man is prone to work harder when he is happier, more weight applied to his feelings results in a much higher amount of effort being displayed. However, neither the man nor the woman seems to benefit from this increased effort, confirming prior results that

moderation is the key for this factor. As a woman's feelings for a man increase, he enters a comfort level that enables him to reduce his efforts. When this effect is magnified, his efforts take a nosedive. Even so, the man and woman still have higher feelings for each other, adding further proof that a couple's feelings for each other play a much bigger role in deciding their fate than the effort the man puts into the relationship.

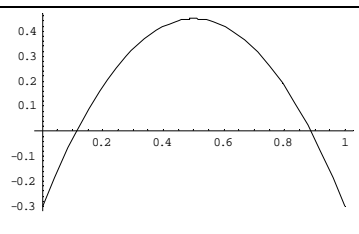
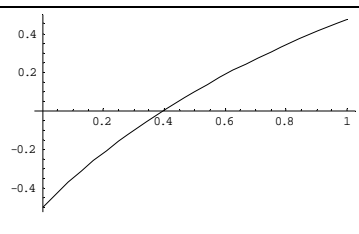
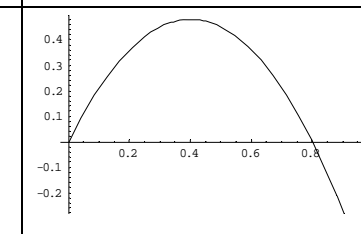
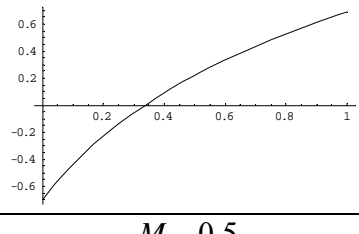
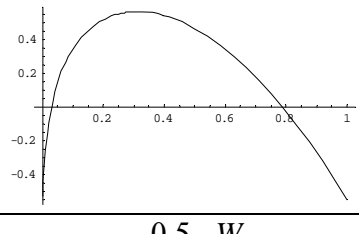
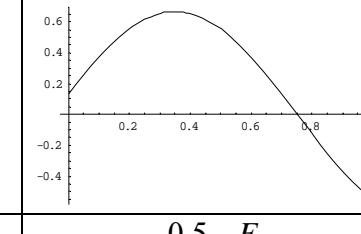
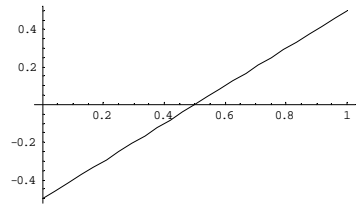
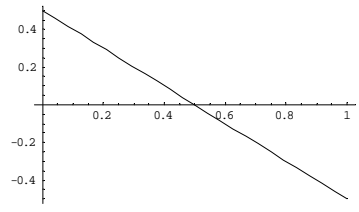
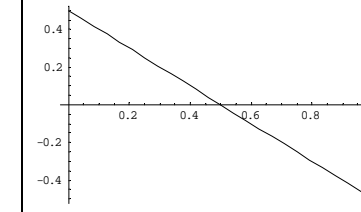
**Conclusion:**

Our model for love using differential equations has proven to be of little use in making accurate forecasts concerning how a relationship will play out in the end; on the contrary, it provides concrete evidence that love is an unpredictable, chaotic event. However, this study has been successful in explaining how some key factors interrelate as a relationship progresses over time. When smitten, men seem to have stronger feelings than women, and they aren't as prone to change those feelings as women are, as evidenced by the average amount of change in the equilibrium analysis (Appendix C). Also, strong changes in feelings have little effect on the man's effort, but changes in effort can have drastic effects on the couple's feelings. Happiness can best be found when a man and a woman share strong feelings for each other, and not a lot of work is required to maintain those feelings. Despite its inherent complexities, this simple statement fully captures the nature of true love.

## Appendix A – Love Prophet I Graphs

	$\frac{-7}{4}M + \frac{3}{4}$	$\sin\left(\frac{-\pi}{12} + \frac{\pi}{0.9}W^2\right)$	$\sin\left(\frac{-\pi}{6} + \frac{4\pi}{3}E\right)$
$\dot{M}$			
	$\sin\left(\frac{-\pi}{12} + \frac{\pi}{0.9}M^2\right)$	$\frac{-3}{2}W + \frac{1}{2}$	$\sin\left(\frac{\pi}{15} + \frac{15\pi}{15}E^{0.42}\right)$
$\dot{W}$			
	$M$	$\sin\left(\frac{13\pi}{12}W^{0.25}\right)$	$\frac{-5}{4}E + \frac{1}{4}$
$\dot{E}$			

### Appendix B – Love Prophet II Graphs

	$3M(1-M) - 0.3$	$\ln(W + 0.606)$	$3E(0.8 - E)$
$\dot{M}$			
$\dot{W}$			
$\dot{E}$			

## Appendix C – Results

	Initial Conditions			Results		
	M	W	E	M	W	E
<b>Love Prophet #1</b>	0.000	0.000	0.000	0.916	0.601	0.625
	0.250	0.250	0.250	0.916	0.601	0.625
	0.500	0.500	0.500	0.916	0.601	0.625
	0.750	0.750	0.750	0.916	0.601	0.625
	1.000	1.000	1.000	0.916	0.601	0.625
	0.500	0.000	0.500	0.916	0.601	0.625
	0.000	0.500	0.000	0.916	0.601	0.625
	0.500	0.500	0.000	0.916	0.601	0.625
	1.000	0.000	0.250	0.916	0.601	0.625
	0.750	0.250	0.750	0.916	0.601	0.625
	0.330	0.670	0.670	0.916	0.601	0.625
	0.138	0.736	0.298	0.916	0.601	0.625

	Initial Conditions			Results		
	M	W	E	M	W	E
<b>Love Prophet #2</b>	0.000	0.000	0.000	0.823	0.780	0.511
	0.250	0.250	0.250	0.823	0.780	0.511
	0.500	0.500	0.500	0.823	0.780	0.511
	0.750	0.750	0.750	0.823	0.780	0.511
	1.000	1.000	1.000	0.823	0.780	0.511
	0.500	0.000	0.500	0.823	0.780	0.511
	0.000	0.500	0.000	0.823	0.780	0.511
	0.500	0.500	0.000	0.823	0.780	0.511
	1.000	0.000	0.250	0.823	0.780	0.511
	0.750	0.250	0.750	0.823	0.780	0.511
	0.330	0.670	0.670	0.823	0.780	0.511
	0.138	0.736	0.298	0.823	0.780	0.511

	Weight Increase		Results (in 10 <sup>-3</sup> )		
	Equation	Term	M	W	E
<b>Equilibrium Analysis</b>	$\dot{M}$	$M$	-37	-4	-9
	$\dot{M}$	$W$	+7	0	+1
	$\dot{M}$	$E$	+41	+3	+9
	$\dot{W}$	$M$	+7	+41	-9
	$\dot{W}$	$W$	-10	-52	+10
	$\dot{W}$	$E$	+7	+38	-8
	$\dot{E}$	$M$	-26	-36	+97
	$\dot{E}$	$W$	+11	+20	-106
	$\dot{E}$	$E$	+1	+1	-6
	<b>Average Change:</b>			16.3	21.7