The Good, the Bad, and the Ugly
The Wolves, the Rabbits, and the Hunters

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Abstract: We will determine the implications of inserting hunting season into predator prey analysis. We will try to discover what results when hunters are reducing wolf population during the winter and summer seasons.

Introduction

The natural cycle that takes place when wolves prey on rabbits is basically consistent and predictable. The different weather seasons that occur through the year are taken into consideration, as well. But basically, the overall sequence for wolves and rabbits is this: The wolves are hungry so they eat a bunch of rabbits, but then there aren't as many rabbits around anymore so the wolves start to die off from starvation. Since the wolf population is lowering then there are less wolves to eat rabbits, hence the rabbit population increases. This increase in food causes the wolf population to rise and then we repeat this cycle for an infinite amount of time. Or can we?

Back before man hunted for sport and the world wasn't so crowded, the beautiful cycle of nature could have lasted forever, with the exception of natural disasters. But the powerful hand of man is now upon us and what will happen if we throw this enemy of Mother Nature into the fight for survival? Could killing a certain percentage of the wolves during the winter and summer seasons mean total annihilation of the wolf population and an immense growth of rabbits? We're researching this project to discover what happens when hunters become involved in Predator-Prey analysis.

Before bringing hunting season in, we need to state some constant differential equations and functions that will represent rabbit growth, wolf growth, and rabbit growth as a function of the four weather seasons. The seasonal function and two differential equations for rabbit and wolf growth are:

$R(x)=a(t)x_1 - 2x_1x_2$

$W(x)=-2x_2 + 5x_1x_2$

$a(t)=3 - 2\sin(t/2)$

This is a modification of the Lotka-Volterra predator-prey model. The variables for "$-2x_1x_2$" and "$5x_1x_2$" are approximations for how many times the wolves and rabbits encounter each other. The terms, "$a(t)x_1$" and "$-2x_2$", are also approximations for how quickly rabbits and wolves reproduce or show prehunting behavior.

The hunting equations are designed so that winter, spring, summer, and fall each have 3 months apiece. For my experiment, I made winter and summer the wolf hunting seasons.

The equations for hunting season are:

$$hun(t)=\begin{cases} .20 & 0<t<3 \\ 0 & 3<t<6 \\ .20 & 6<t<9 \\ 0 & 9<t<12 \end{cases}$$

where t is measured in months. Every hunting season, the hunters will kill 20% of the wolf population.

With hunting season coinciding with winter and summer there are sharp rises and falls in the growth of both the rabbits and wolves, followed by extended times of low population. The graph representing the growth of rabbits during the course of 3 years is similar to that of the wolves.

The growth of rabbits with respect to months is:

rabbits(x10,000)

time(months)

Figure 1

The growth of wolves with respect to months is:

wolves(x10,000)

time(months)

Figure 2

It is seen that at the time that wolf population begins to increase rapidly on one graph, the rabbit population begins to decline. Also noticeable is that rabbits always have a steady incline with a sharp decline while the wolves have vice versa.

At times, as in the first months 1-3 and 12-15, there has been lesser growth in the wolf population. This could best be attributed to certain times (winter) where rabbits were struggling to survive on their own which meant smaller food population, and at the same time the hunters were killing off many of the wolves. Small growths within the rabbit graphs could be attributed to basically the same idea, that wolves are eating all of them while they're struggling to survive in cold temperatures as it is.

At other times, wolf population has an immense growth, which can be a result of being hunted in a previous warm season (summer) and giving the rabbits plenty of time to live, reproduce, and have their food grow beyond proportion. This also explains why the rabbits sometimes grew large in population.

Combining these two graphs shows how when one population grows the other diminishes. But, because of hunting season killing 20% of the wolves half the time, this graph isn't the usual cycle of nature. It's more of a spiral that grows wider in radius and then smaller on a very inconsistent basis. (much like the indivudal graphs of wolves and rabbits oscillate on an inconsistent basis.)

(wolves)

(rabbits)

Figure 3

This graph begins with the wolves having a population of 2 (relative to actual population) and it increases by spiraling in a counter-clockwise motion. As wolves increase, rabbits are shown decreasing.

Though the 20% of dead wolves during hunting season was probably a great help for the rabbits, we didn't think it was enough for the little furry animals. Therefore, I decided to raise the quota up to 70%. The hunters, during hunting season, were now able to kill 250% more wolves than they could before. In doing this I discovered that the wolf population decreased from 15 to 10, which is a very large drop and seemed quite acceptable. What was less encouraging, though, was the rabbit population:

rabbit

time

Figure 4

I figured the rabbits to have had an enormous growth as a result of the decrease in wolves, but the highest point actually dropped 2 points from the previous when only 20% of the wolves were being hunted. It almost seemed as though hunting season may decide both growth rates in a proportionate manner. To test this idea, we tried making the hunting quota 50% and 10% to attempt to find a pattern.

But, there seems not to be much of a pattern, for when it was lowered to 50%, the highest point raised to about 3.5 for the rabbits;

rabbit

time

Figure 5

while wolves stayed at the same population of 10.

Strangely enough, when the quota lowered to 10%, both the wolf and the rabbit population increased. The wolf by 2 points and the rabbits by 1 point,

rabbit

time

Figure 6

wolf

time

Figure 7

Evidently, there seems to be no pattern arising in the raising or lowering of the quota for hunting in proportion to rabbit and wolf population, but a more extensive study may provide one. And, for some odd reason, the wolf population still decreased from 14 to 12 when less wolves were supposedly being killed.

After this bizarre activity, I decided to just kill off 100% of the wolves, total extinction. I assumed that surely this would result in an asymptote in the graph where rabbits increase into infinity because, hypothetically, there are no more animals around to eat them, and a spot where the wolf function would vanish into zero.

rabbit

time

Figure 8

wolf

time

Figure 9

All that happened was a decrease in both populations! At first, I wondered if I did something wrong in my calculations

and then I realized it wasn't us: the computer was understanding the problem incorrectly. In reality, the wolves would all be dead if 100% of them were allowed to be hunted. In reality one wolf equals one unit on a graph in our minds. But in the computers mind, there is no concept of 1 unit equaling one wolf. Hence, .2 wolf or .7 wolf is found to be logical in a computer. Therefore, when the population got below 1 whole wolf, there weren't any more wolf to kill according to the hunting function (hun(t)). With no more hunters, this fraction of a wolf could lay low for a while until hunting season was over and could start eating and reproducing again. With the way that the differential equations and functions are setup, there will be no way for either predator or prey to become extinct.

Conclusion

Modeling predator-prey analysis makes it easier to track different patterns of nature. Throwing hunters into the equation is one of many ways to disrupt the cycle. Weather affecting rabbits along with hunters affecting wolves can create predictable actions in the growth of either animals, yet conclude bizarre results in the long run. An extended project for future work may very well be discovering why do both the rabbit and the wolf population decline overall if more wolves are being killed. Other ideas could be to switch hunting seasons by making it 1 out of the 4 seasons, 4 out of the 4 seasons, or alternating seasons. Predator-Prey analysis is a project with as many different directions as stars in the sky.