Abstract
A young baseball player stacks $n$ baseball hats by each door to his home. Each time he leaves the house to go practice, he grabs a hat from the stack by the door he exits; when he returns to his home after practice, he leaves his hat on the stack by the door he enters. In our problem we consider how many times, on average, the baseball player will go out to practice and back into his house before the stack of baseball hats by the door he exits runs out. We begin with an examination of two doors starting with $n$ hats by each to determine a formula that calculates how many cycles the boy will run through before he goes to grab a hat as he leaves the house, evaluate the variance surrounding our average, introduce the examination of two doors starting with a stack of baseball hats by the door he exits runs out. We then use systems of equations and a C++ simulation to find the average number of times Py goes out and back into his house for each case.

The Question:
• How many times, on average, will Py go out to practice and back into his house before his stack of baseball hats by the door he exits runs out?

Process:
• We use Markov Chains to illustrate each state and the probability of moving from one to the next, drawing arrows from the original state to the next.
• We translate the probabilities found in the Markov Chains into transition matrices. The rows represent the starting state and the fraction displayed in each position is the probability that the row-heading state will move to that column’s state.
• Therefore, the sum of each row should equal 1. Our matrices will always be square and the last row will always be filled with zeroes except for the last entry which will be a 1, because the probability of leaving the absorbing state is 0 and the probability of staying in it is 1.
• We create Markov chains and matrices for the first several cases, starting with $n = 1$ hat by each door.
• We then use systems of equations and a C++ simulation to find the average number of times Py goes out and back into his house for each case.

Results and Discussion
• Theorem 1: Given two doors with $n$ hats by each, the average number of times it will take before there are zero hats by the door the boy exits is: $H_n = 2n^2 + 2n + 1$.
  • We prove this theorem using equations established by the $n$ hats matrix, systems of equations, and then piecing it back together.
  • We use Mathematica to determine the approximate range for each trial run (figure 4).

Areas for Future Research: further extrapolating the case with three doors; the case with $n$ doors and only one hat to start at each; the case where we lose (or gain!) a hat; the case where we start with different numbers of hats by each door; why an induction proof also works; and so on.

Figure 1. Picture this: Py entering and exiting his house, picking up a hat by the door he exits and dropping off a hat by the door he reenters. A state (partition), $S$, is an arrangement of the hats by the doors. In this picture, the state is $(a, b)$. The absorbing state signifies the end of the problem. We enter the absorbing state when we try to exit a door that has zero hats by it. When we are in the absorbing state our probability of staying in that state is 1. Given state $S$, $H_n$ is the average number of times it will take Py to travel out of the house and back in before running out of hats.

Figure 2. The Markov Chain and corresponding matrix for the case with $n=1$ hats.

Figure 3. The matrix for the case with $n$ hats.

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